International CAE Conference
Pacengo del Garda (Verona) - Italy, 19 - 20 October 2015

Simulation and Control of Multi-Agent Systems: A Minimal Evacuation Time Problem

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Motivation and Goals

- *Realistic Simulations* and *Optimal Control* of multi-agent systems require the reiterative solution of a large system of ODEs, which drives usually to unaffordable computational costs.
- We *approximate the problem with a kinetic PDE* and simulate its constrained evolution by means of *Binary Interaction algorithms*, which are able to reduce drastically the computational cost.
- Applications find place in several areas: *engineer, biology, robotics, computer graphics . . .*
- In [1] we are concerned with the multi-scale modeling, control and simulation of a crowd leaving an unknown area.

Problem setting

- We model human crowd leaving an *unknown environment*, Ω, under limited visibility. Due to their lack of information about the positions of exits, agents need to explore the environment first.
- Our aim is to *control the emergent dynamic* through the action of *few informed agents*, not recognized by the rest of the crowd.
- We consider N followers and M leaders whose interactions are described by

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= S(x_i, v_i) + \sum_{j=1}^{N} F(x_i, v_i, x_j, v_j) + \sum_{l=1}^{M} F(x_i, v_i, y_l, w_l), \\ \dot{y}_k &= w_k = \sum_{j=1}^{N} K(y_k, x_j) + \sum_{l=1}^{M} K(y_k, y_l), + u_k \end{aligned} \qquad (1)$$

  where S represents a self-propulsion term and F, K the interaction kernels accounting the social forces of the dynamic: *alignment, repulsion and attraction, low visibility*.
- u_k is the *control* chosen as a solution in the set of *admissible controls* U_{adm} of

$$\min_{u(\cdot)\in U_{\text{adm}}} \{t > 0 \mid x_i(t) \notin \Omega, \quad \forall i = 1, \dots, N\}, \qquad \text{subject to (1).}$$

Simulation of the microscopic model

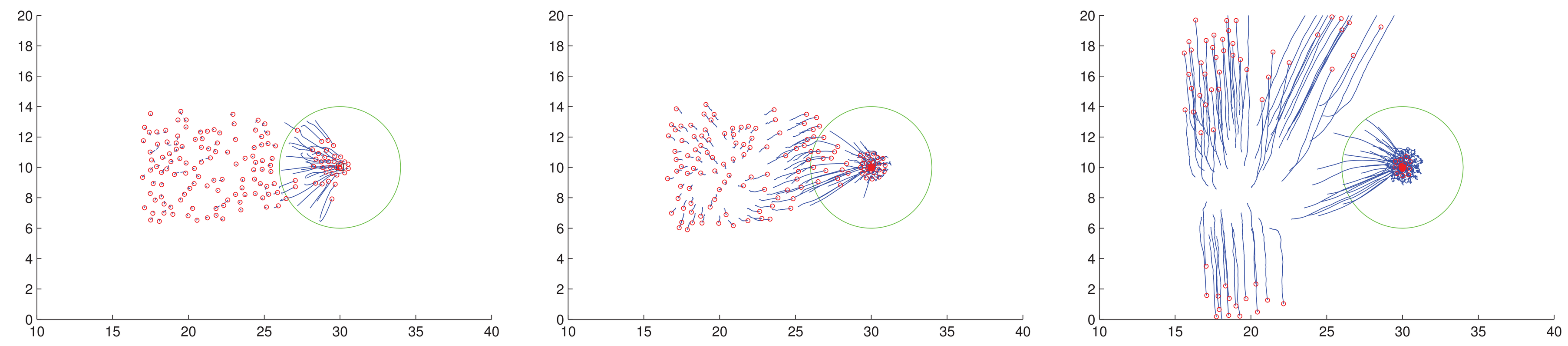

Figure : The exit is a point located at $E = (30, 10)$ which can be reached from any direction and it is visible inside the green circle. $N = 100$ Followers are initially randomly distributed in the domain $[17, 29] \times [6.5, 13.5]$ with velocity $(0, 0)$. Without leaders the total mass is not evacuated, farthest people split in several but cohesive groups with random direction and never reach the exit.

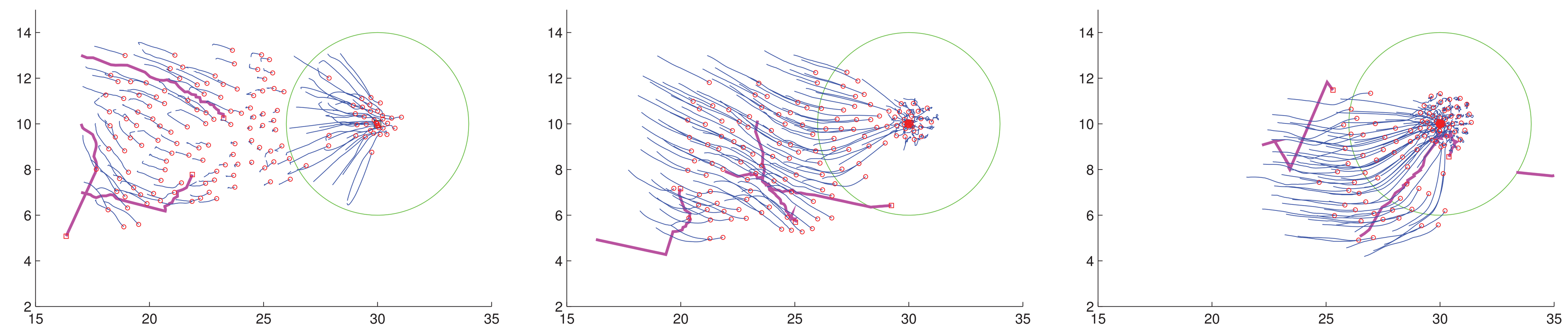

Figure : The optimal strategy u_k is obtained through a modified compass search method. With $M = 3$ leaders the optimal strategy u_k, prescribes that leaders *divert* some pedestrians from the right direction, so as not to steer the whole crowd to the exit at the same time. In this way congestion is avoided and pedestrian flow through the exit is increased.

Mesoscopic model

- When the number of follower, N, is large, a microscopic description of both populations is no more a viable option. Thus we consider the evolution of the distribution of followers, denoted by $f(x, v)$, together with the leaders, with empirical distribution $g(y, w)$.
- We assume that these densities satisfy the following coupled ODE-PDE system

$$\begin{aligned} \partial t f + v \cdot \nabla_x f &= Q_F(f, f) + Q_L(f, g) \\ \dot{y}_k = w_k &= \int K(y_k, x) f(x, v) dx dv + \sum_{\ell=1}^{M} K(y_k, y_\ell) + u_k, \end{aligned} \qquad (2)$$

  where $Q_F, Q_L(f, g)$ represents two Boltzmann-like operators accounting the instantaneous rate of change of the particles' density.

Binary Interaction algorithms

- In order to simulate the dynamic of (2), we relay on Binary interaction algorithms.
- This approach is based on Monte-Carlo procedure, which allows to solve efficiently the interaction kernels, see [2, 3] for a detailed description.
- The main advantages with respect to standard methods are: the linear cost for the evaluation of the interaction kernels Q_F, Q_L, and a full meshless algorithm.

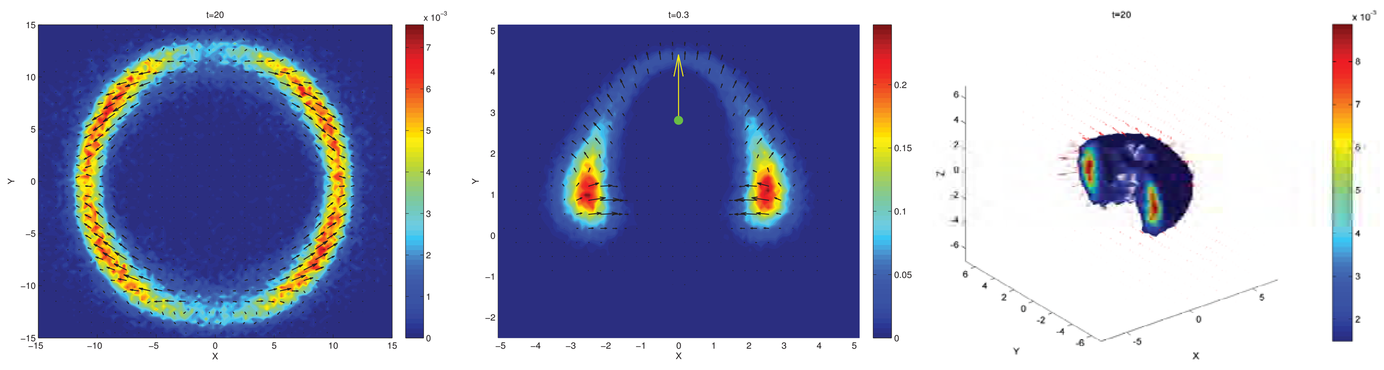

Figure : Simulations for the kinetic description of multi-agent systems through BI algorithms.

Simulations of the mesoscopic model

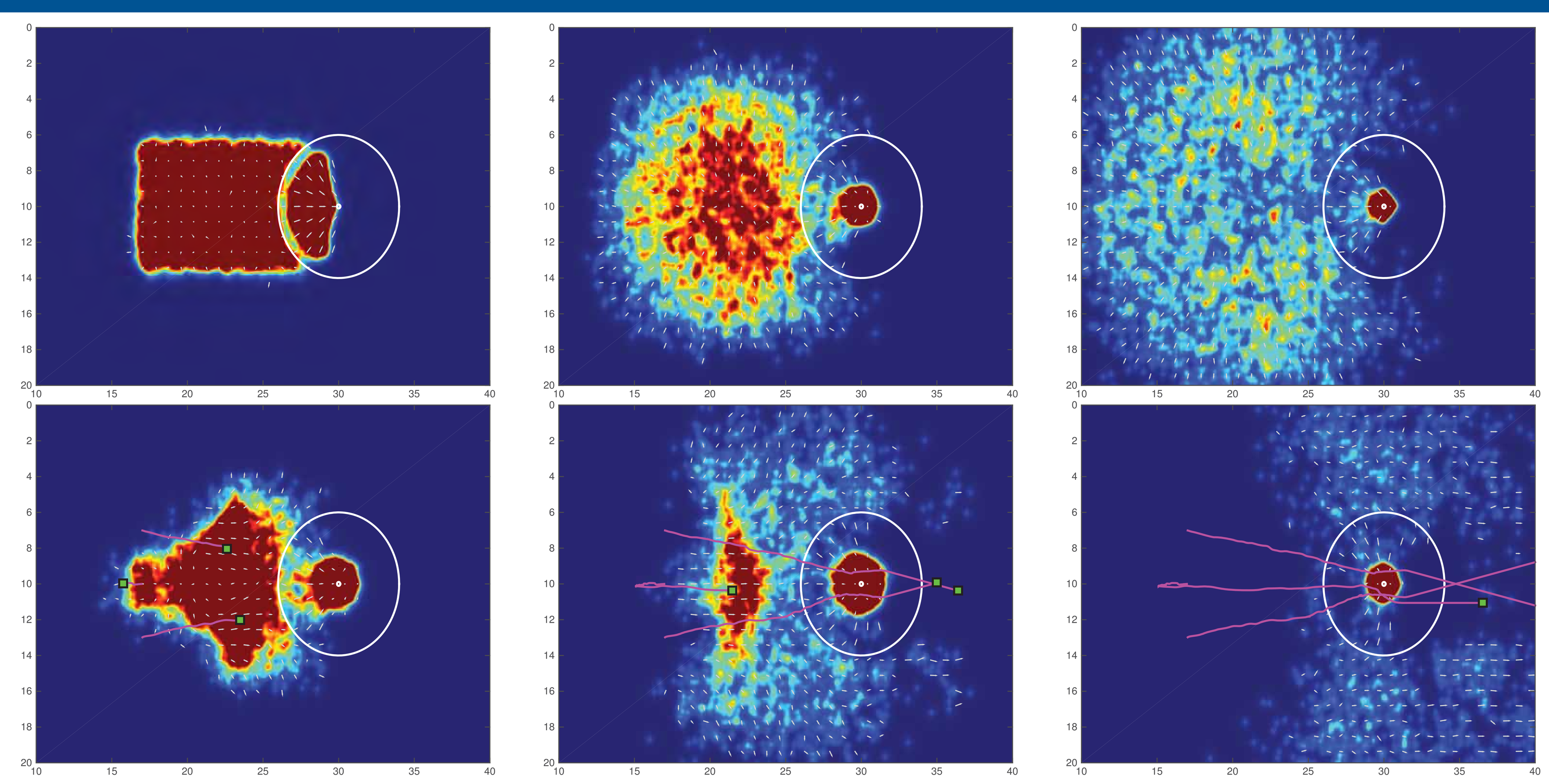

Figure : First row: no leaders. Second row: three leaders, optimal strategy u_k (computed via compass search methods).

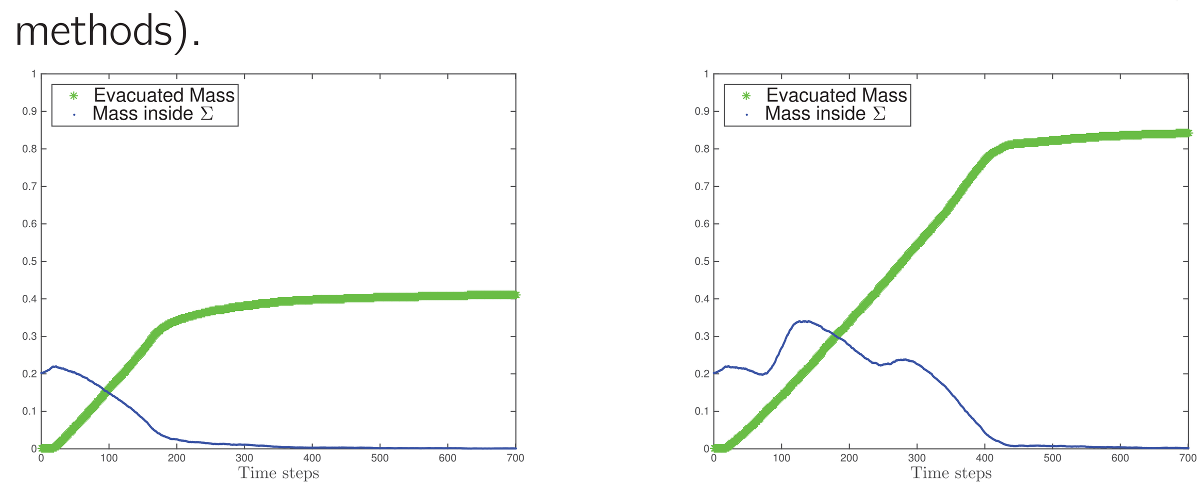

Figure : The blue line represents the mass concentration around the exit and green line the percentage of total evacuated mass, as functions of time. Case without leaders (left), total evacuated mass: 41.2%. Case $M = 3$ leaders (right) percentage of evacuated mass 85.2%. Optimal strategies avoid congestions at exits.

References

[1] G. Albi and M. Bongini, E. Cristiani and D. Kalise, *Invisible Control of Self-Organizing Agents Leaving Unknown Environments*, Preprint No. IGDK-2015-09., (2015).

[2] G. Albi and L. Pareschi, *Binary interaction algorithms for the simulation of flocking and swarming dynamics*, Multiscale Model. Simul., 11 (2013), pp. 1–29.

[3] L. Pareschi and G. Toscani, *Interacting multi-agent systems. Kinetic equations & Monte Carlo methods*, Oxford University Press, USA, 2013.

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